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REPORT DOCUMENTATION PAGE

Form Approved
OMB NO. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	~~April 1, 1997~~	Final 1993-1995 (3 years)

4. TITLE AND SUBTITLE
Signal Detection, Target Tracking and Differential Geometry Applications to Statistical Inference

5. FUNDING NUMBERS
DAAH04-93-G-0030

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(ES)
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8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING / MONITORING AGENCY REPORT NUMBER
ARO 30529.96-MA

11. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12 b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

Signal detection and target tracking. A novel method known as polynomial rooting approach is proposed to obtain estimates of frequencies, amplitudes and noise variance of two-dimensional exponential signals. The consistency and asymptotic normality of the least squares estimators in a multidimensional exponential signal model are established. Significant contributions have been made to the design of observations, efficient estimation of target positions almost continuously in time and establishing correct association between estimates of target positions made at different time points.

M-estimation. A unified theory of robust estimation is developed using the difference of two convex functions as the discrepancy measure.

Quantile regression. The concept of a quantile regression function is introduced as the conditional mean of the response variable at the u-th quantile of the independent variable and applied to problems of inference when the independent variable in different samples are not comparable.

14. SUBJECT TERMS	15. NUMBER IF PAGES
Signal detection, Target tracking, M-estimation, Quantive regression, Score statistic, Probability theory	
	16. PRICE CODE

17. SECURITY CLASSIFICATION OR REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Enclosure 1

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18

Foreword

This is the Final Report of research activities at the Center for Multivariate Analysis, Statistics Department, Pennsylvania State University, under support from the Army Research Office, Contract No. DAAH04-93-G-0030, during the three-year period 1993-95.

The results of research are reported in various technical reports issued from the Center for Multivariate Analysis and papers published in refereed journals and conference proceedings.

The research was undertaken by a team of senior scientists and post doctoral research associates, who are all supported by the grant from ARO during the period under review. The work was done under the direction of the principal investigator, C.R. Rao, Eberly Professor of Statistics and Director of the Center for Multivariate Analysis. The senior collaborators were M.B. Rao, Professor of Statistics at the North Dakota State University, D.N. Shanbhag, Associate Professor at the University of Sheffield, U.K., Rahul Mukerjee, Professor at the Institute of Management, Calcutta, India, D. Kundu, Associate Professor, Institute of Technology, Kanpur, India and L.C. Zhao, Professor at the University of Science and Technology, Beijing, China, all of whom have numerous research publications to their credit.

The research was conducted on a broad spectrum of areas of applications of statistics in defense oriented problems. Special mention may be made of contributions to signal detection, estimation and tracking of multiple moving objects, and properties of estimates of two and three dimensional superimposed exponential signals. Another area of interest is robust estimation of unknown parameters, where a unified theory is developed using the difference of two convex functions as the discrepancy measure. In addition some new limit theorems in probability and characterization of probability distributions are obtained, and their applications are discussed.

I wish to thank the ARO for the financial support which enabled the Center for Multivariate Analysis to put together a strong team of research staff to work on defense oriented problems.

Center for Multivariate Analysis
Penn State
March 25, 1997

C.R. Rao

Table of Contents

1. TECHNICAL OBJECTIVES AND SUMMARY OF RESULTS

1.1 Technical Objectives and Motivation

Some problems in signal detection, estimation and tracking of moving objects have been investigated. The main object is to derive efficient methods of estimation with a minimal number of assumptions on the noise components and to develop suitable computational algorithms to implement the methods.

A general theory of robust estimation of unknown parameters is proposed by minimizing a loss function which is the difference of two convex functions. Such a loss function enables the use of elegant properties of convex functions in deriving general results characterizing the properties of estimators. Since any function can be approximated by the difference of two convex functions, our approach provides a unified theory of robust estimation under a minimal set of conditions. There is a growing literature on the computational aspects of minimizing the difference of two convex functions. This will be an additional advantage on the computational side in the application of M-estimation using the difference of two convex functions as a discrepancy measure.

Characterization of probability distributions play an important role in modelling some stochastic mechanisms for data analysis and statistical inference. A number of results have been established on the characterization of some distributions and some practical applications based on such distributions have been considered.

Some new limit theorems in probability theory have been established and used in establishing consistency of estimators.

In practice, there are many situations in which observations are censored. A satisfactory theory for the analysis of a regression model when the dependent variable is censored has been developed.

1.2. Approach

In order to study the above problems some new tools have to be forged.

One is the discovery of a new convergence theorem in probability which can be applied in a variety of situations to establish asymptotic consistency of estimators (TR 95-19). The result is as follows:

For any fixed k let $X(t_1,\ldots,t_k)=X(\underset{\sim}{t})$, when $t=(t_1,\ldots,t_k)$ be an iid sequence of real random variable for $t_i=1,2,\ldots,$; $i=1,\ldots,k$ with mean zero and

$$E|X(\underset{\sim}{t})|\log^k(1+|X(\underset{\sim}{t})|)<\infty,$$

then

$$\sup_{\theta_1,\ldots,\theta_k} \left|\frac{1}{n_1\ldots n_k}\sum_{t_1=1}^{n_1}\ \sum_{t_k=1}^{n_k}\ X(\underset{\sim}{t})\ \cos(t_1\theta_1)\ldots\ \cos(t_k\theta_k)\right|$$

converges to zero almost surely as $\min(n_1,\ldots,n_k)\to\infty$.

We extended some theorems of Rockefellor on convex functions to establish the result that any function can be approximated by the difference of two convex functions.

An important tool known as the Integrated Cauchy Functional Equation, which is a generalization of the Choquet-Deny theorem on a functional equation, has been introduced in proving some characterizations of probability distributions. The research done on the Integrated Cauchy Function Equation led to the publication of the book Choquet-Deny Type Functional Equation and its Application to Stochastic Models by Wiley in 1994 under the joint authorship of C.R. Rao and D.N. Shanbhag.

A new law called the Strassen's law of the iterated logarithm for the empirical cumulative regression function has been established and applied to problems arising in testing differences between populations when the measurements are not strictly comparable.

It is shown that the expected value of the harmonic mean of two positive random variables is not greater than the harmonic mean of their expected values. This new inequality is crucial in examining conditional inference based in ancillary statistics advocated by R.A. Fisher.

2. SIGNIFICANT ACCOMPLISHMENTS

2.1 Signal detection and estimation:

(Tech.Repts.93-05, 93-24, 93-27, 94-04, 94-14, 94-16, 94-17, 94-18, 95-02, 95-16, 95-17, 95-18, 95-19, 95-20, 95-22)

The problem of maximum likelihood estimation of frequencies, amplitudes and noise variance of two-dimensional exponential signals is considered. A polynomial rooting approach is proposed to obtain the estimates of the two dimensional frequencies. Strong consistency and limiting distributions are established for the estimates of the parameters. Furthermore, the covariance of the limiting distribution attains the Cramer-Rao lower bound. A new computational algorithm is proposed for the above problem, incorporating the method for estimating the number of signals.

The consistency and asymptotic normality of the least squares estimator are derived for a particular non-linear regression model which does not satisfy the standard sufficient conditions. The results are applied to signal detection and estimation problems.

High resolution parameter estimation is an important problem in signal processing applications. Such applications include direction of arrival (DOA) estimation for narrow band signals emitted by multiple sources and received by sensor arrays. Some proposed methods of estimation like TLS-ESPRT and MUSIC give satisfactory results. The performance of these methods has been significantly improved by using the data vector as well as its conjugate and making suitable modifications.

The consistency of the least squares estimates in a multidimensional exponential signals model is established under weaker assumptions than those existing in the literature by using a new convergence theorem in the theory of probability.

The consistency and asymptotic normality of the least squares estimator are derived for some types of non-linear time series models under the assumption that the errors are independently and identically distributed random variables each with zero mean and finite variance.

A number of problems in the estimation of superimposed exponential signals have been investigated.

A method has been proposed for estimating the parameters of undamped exponential signals when observations are missing. The consistency of the method is established and the behaviour of the estimates in small samples has been studied through simulation.

There is considerable literature on the estimation of parameters in the undamped exponential signals model

$$y_t=\alpha_1 \exp(i2\pi\omega_1 t)+\ldots+\alpha_m \exp(i2\pi\omega_m t)$$
$$t=1,\ldots,n.$$

Some of the open problems connected with the above model have been investigated. The consistency of the least squares estimators has been established under a restriction on the parameter space. The asymptotic distribution of the estimates is obtained without the assumption of normality of the distribution of the error terms. On the computational side, to obtain the least squares estimators, a nonlinear eigen value method is proposed which seems to work well in practice.

2.2 Quantile regression function

(Tech.Repts. 93-26, 93-28, 94-01, 94-08, 94-10)

A new line of research is initiated by introducing the concept of a quantile regression function.

Let (X,Y) be a bivariate random variable with $E|Y|$ finite and denote by $m(x)=E(Y|X=z)$, the regression of Y on X. Further let F(x) be the distribution function of X. The quantile regression function of Y on X is defined by

$$r(u)=m(F^{-1}(u)),\ 0\leq u\leq 1$$

and the cumulative quantile regression function by

$$M(u)=\int_0^u r(t)\,dt.$$

By considering the empirical distribution of (X,Y) based on a sample, $(X_1,Y_1),\ldots,(X_n,Y_n)$, we can estimate M(u) by

$$M_n(u)=n^{-1}\sum_{i=1}^{[nu]} Y_{(i)}$$

where $Y_{(i)}$ is the value of Y associated with $X_{(i)}$, the *i*-th order statistic of X. The asymptotic properties of $M_n(u)$ have been studied, such as convergence to true value and the Strassen's law of large numbers.

Tests are constructed for comparing the quantile regression functions in two populations. This is useful in situations where the independent variable is not comparable, like income of individuals in different countries.

2.3 Robust estimation

(Tech. Repts. 93-06, 93-11, 93-13, 94-06, 95-03)

Several alternatives to least squares estimation of unknown parameters based on minimizing the sum of squared deviations of observations from expected valued have been suggested. Instead of squared deviation, a suitably chosen discrepancy function which does not increase rapidly with the argument is shown to provide more stable estimators of the parameters. Such a method of estimation is called M-estimation. There is, however, no satisfactory theory establishing the properties of such M-estimators under simple conditions. When the discrepancy measure is a convex function, an elegant theory is available under a minimal set of conditions. Recently the theory is extended to a discrepancy measure which is the difference of two convex functions under the same minimal conditions. It is also established that any discrepancy measure can be approximated by a difference of two convex functions. Thus a unified and an elegant M-estimation theory is obtained.

There is a growing literature on the computational aspects of minimizing the difference of two convex functions. This will be an additional advantage on the practical side in the application of the theory of M-estimation based on a discrepancy measure which is the difference of two convex functions.

The theory of M-estimation of regression parameters is extended to the case where the dependent observation is censored. Under some mild conditions, the weak consistency and asymptotic normality of the estimates, obtained by the least absolute deviations method, are obtained. These results open up a wide area of research of practical importance, as in many situations the dependent variable is censored.

2.4 Score statistic

(Tech. Repts. 93-01, 93-07, 95-04)

The score statistic for testing simple and composite hypotheses as an alternative to Wald and likelihood ratio tests was suggested by Rao in 1949. Since then there has been considerable research on examining the relative merits of these statistics.

The paper by Rao has been reproduced as a separate Chapter in the book Breakthroughs in Statistics: 1890-1989 (during the last 100 years). A separate session on "50 years of Rao's score test" is being organized at the joint statistical meetings of IMS, ASA, BS, SSC at Anaheim in August 1997.

Some new properties of score statistics have been established. Several Bartlett-type adjustments of the score statistic have been considered and their relative powers have been examined under contiguous alternatives, with reference to the criteria of maximinity and average power. Conditions for approximate frequentist validity of posterior credible sets based on the

score statistic have been derived in the multiparameter case. In the process, explicit expressions are given for the posterior quantiles of the score statistic and a posterior Bartlett-type adjustment is suggested for the same.

2.5 Probability theory

2.5.1 Characterization of distributions

(Tech. Repts. 93-29, 93-30, 94-13, 95-01, 95-15, 95-26, 95-30)

Characterization of probability distributions through properties of sample statistics plays an important role in statistical inference. An important tool for such studies is the integrated cauchy functional equation introduced by Rao and Shanbhag. Some extensions of this equation have been made to provide characterizations of the exponential distribution based on the properties of order statistics and record values. These results are useful in survival analysis and reliability theory.

2.5.2 Limit theorems

(Tech. Repts. 93-20, 93-21, 93-23, 94-08, 94-09, 94-10, 95-07)

L-statistics play an important role in robust statistical inference. Marcinkiewicz-Zygmund type strong laws of large numbers and bounded Hartman-Wintner type law of the iterated logarithm are established for linear combinations of order statistics with smooth score functions under a minimal moment condition. A law of the logarithm for weighted sums of order statistics is also established.

The law of iterated logarithm is established for random geometric series, $\xi(\beta)=\sigma\beta^n\epsilon_n, \beta\in(0,1)$ when $\{\epsilon_n: n\geq 0\}$ is an iid Bernoulli sequence.

An open problem posed by Hu and Weber on the limit behavior of $S(n)/\sqrt{2n\log n}$, where S_n is the sum of iid Banach space valued random variables, is solved.

2.5.3 Risk models and probability of ruin

(Tech. Repts. 94-11, 94-12)

Some probability models for claim amounts from the insured are considered and the probability of ruin of the insurer is computed in each case. A model of particular interest is the mixture of exponentials for the distribution of claim amounts and Poisson for the number of claims. An alternative distribution, which is a mixture of Poisson distributions, is also considered for the number of claims.

Appendix A

Research Personnel of the Center for Multivariate Analysis (1993-95)

1. C.R. Rao (Principal Investigator)

Eberly Professor of Statistics, Statistics Department, Pennsylvania State University.

2.R. Mukerjee

Professor at the Indian Institute of Management, Calcutta, India. Has a Ph.D. in Statistics and is the author of 150 research papers, most of which are published in refereed journals.

3. L.C. Zhao

Professor at the University of Science and Technology, China. Has a Ph.D. in Mathematical Statistics and is the author of 100 research papers published in refereed journals.

4. M.B. Rao

Professor of Statistics at the North Dakota University, Fargo. Has a Ph.D. in Statistics and is the author of 85 research papers, most of which are published in refereed journals.

5. D.N. Shanbhag

Reader in Statistics at the University of Sheffield, U.K. Has a Ph.D. in Statistics and is the author of one book and 84 research papers, most of which are published in refereed journals.

6. D.N. Kundu

Associate Professor, Indian Institute of Technology, Kanpur. Author of 50 papers on signal detection, estimation and target tracking.

7. Bin Zhou

Postdoctoral Fellow. Has a Ph.D. in Electrical Engineering. Author of 10 research papers in signal processing and multitarget tracking.

Appendix B

Honors received 1993-95

C.R. Rao, the principal investigator received the following honors during the period under report, 1993-95.

1994

1. D.Sc. (Hon. causa), Slovak Academy of Sciences, Slovakia.
2. Elected Fellow of American Association for the Advancement of Science (AAAS).

1995

1. Dr.oec.publ (Hon. causa), University of Munich, Germany.
2. D.Sc. (Hon. causa), S.V. University, Tirupati, India.
3. D.Sc. (Hon. causa), University of Barcelona, Spain.
4. Elected: Member of National Academy of Sciences, USA.
5. Elected: Honorary Fellow of the Institute of Combinatorics and its Applications.
6. Proclaimed by the Mayor as a Distinguished Citizen of the State College Community and deserving of the highest regards of his friends and neighbors there in.

Appendix C

List of papers published in refereed journals and conference proceedings

1.1 C.R. Rao (Principal Investigator)

1993

1. (with Lu Zhang and L.C. Zhao). Multiple target angle tracking using sensor array outputs. *IEEE Transactions on Aerospace and Electronic Systems,* 29, 268-271.

2. (with L.C. Zhao). Asymptotic behavior of maximum likelihood estimates of superimposed exponential signals. *IEEE, ASSP* 41, 1461-1463.

3. (with M.S. Srivastava and Y. Wu). Some aspects of quality control methods. In *Quality through Engineering Design (Advances in Industrial Engineering* 16), Elsevier, pp. 21-32.

4. (with R.L. Fountain). Further investigations of Berkson's example. *Communications in Statistics, Theory and Methods,* 23, 613-629.

5. Current trends of research in statistics: small sample asymptotics, resampling techniques and robustness. ARO Report 93-1 *Transactions of the Tenth Army Conference on Applied Mathematics and Computing,* pp. 195-220.

6. (with E.B. Fosam and D.N. Shanbhag). Comments on some papers involving the integrated Cauchy function equation. *Statistics and Probability Letters,* 17 299-302.

7. (with Z.D. Bai and L.C. Zhao). MANOVA type tests under M-theory for the standard multivariate linear model. *J. Statistical Planning and Inference,* 36, 77-90.

8. Statistics must have a purpose: The Mahalanobis dictum. *Proc. ISI 49th Session,* Firenze, IN01.1, 21-36.

9. (with G.J. Babu). Bootstrap methodology. In *Handbook of Statistics 9: Computational Statistics,* Ed. C.R. Rao. North Holland, 627-661.

10. (with C.R. Sastry and Bin Zhou). Some recent contributions to multitarget tracking. *Multivariate Analysis: Future Directions.* Ed. C.R. Rao. North Holland, 319-346.

11. (with Paula M. Caligiuri). Analysis of ordered categorical data through appropriate scaling. In *Handbook of Statistics 9, Computational Statistics,* Ed. C.R. Rao. North Holland. 521-534.

12. (with Bin Zhou). Closed form solution to the estimates of direction of arrival using data from an array of sensors. In *Handbook of Statistics 10: Signal Processing,* Eds. N.K. Bose and C.R. Rao. North Holland, 741-754.

13. (with D.S. Tu and Z.J. Liu). Strong representations for kernel estimates of conditional medians. In *Statistics and Probability: A Raghu Raj Bahadur Festschrift.* Eds. J.K. Ghosh, S.K. Mitra, K.R. Parthasarathy, B.L.S. Prakasa Rao, Wiley Eastern Ltd., 381-394.

14. (with Lu Zhang and L.C. Zhao). A new algorithm for multitarget angle tracking. *IEE Proceedings, Part F (England),* 140, 335-338.

15. J.B.S. Haldane: a polymath in the Indian context. J.B.S. Haldane centenary lecture in *Human Population Genetics* (Ed. P.P. Majumdar). Plenum Press, 1-6.

16. (with Paula M. Caligiuri). On scaling of ordinal categorical data. In *Multivariate Analysis: Future Directions 2* (eds. C.M. Cuadras and C.R. Rao). North Holland, 97-112.

17. (with L.C. Zhao). Asymptotic normality of LAD estimator in censored regression models. *Mathematical Methods of Statistics,* 2, 228-239.

18. (with Z.D. Bai and Z.J. Liu). On the strong consistency of M-estimates in linear models under a general discrepancy function. In *Handbook of Statistics*, Vol 11, *Econometrics,* Eds. G.S. Maddala, C.R. Rao, H.K. Vinod, North Holland, 381-392, 1993.

19. (with L.C. Zhao and B. Zhou). A novel algorithm for 2-dimensional frequency estimation. *The 27th Asilomar Conference on Signals, Systems & Computers,* Vol 1 of 2, 199-202.

20. (with W. Schaafsma, G.M. Steerneman and G.N. van Vark). Inference about the performance of Fisher's linear discriminant function with applications to signal detection. *Sankhyā* B, 55, 27-39.

21. (with D.N. Shanbhag). A stability theorem for the integrated Cauchy functional equation. *Gujarat Statistical Review,* Professor Khatri Memorial Volume, 175-184.

22. (with Z.D. Bai and L.C. Zhao). Manova tests under a convex discrepancy function for the standard veultivariate linear model. *J. Statistical Planning and Inference* 36, 77-90.

1994

23. Some statistical problems in multitarget tracking. In *Statistical Decision Theory and Related Topics V,* Eds. S.S. Gupta and J. Berger, Springer-Verlag, 513-522.

24. (with Lu Zhang and L.C. Zhao). Multitarget angle tracking, an algorithm for data association. *IEEE Trans. Signal Processing* 42, 459-462.

25. (with Rahul Mukerjee). Tests based on score statistics: Power properties and related results. *Mathematical Methods of Statistics* 3, 46-61.

26. (with C.R. Sastry and B. Zhou). Tracking the direction of arrival of multiple moving targets. *IEEE Trans. Signal Processing* 42, 1133-1144.

27. (with L.C. Zhao and Bin Zhou). Maximum likelihood estimation of two-dimensional superimposed exponential signals. *IEEE Transactions of Signal Processing* 42, 1795-1802.

28. (with T. Sapatinas and D.N. Shanbhag). The integrated Cauchy functional equation: some comments on recent papers. *Advances in Applied Probability* 26, 825-829.

29. Statistics: an essential technology in environmental research and management. *Environmental and Ecological Statistics* 1, 7-19.

30. (with L.C. Zhao). Berry-Easseen bound for finite population t-statistic. *Prob. and Statist. Letters* 21, 409-416.

31. Some statistical problems associated with signal processing. *Conference Record, 28-th Asilomar Conference on Signals, Systems and Computers* Vol 1, 2-4.

1995

32. (with J.K. Baksalary and A. Markiewicz). Admissible linear estimation in the general Gauss-Markoff model with respect to an arbitrary quadratic risk function. *J. Statist. Planning and Inference* 44, 341-347.

33. (with X.J. Liu). Asymptotic distribution of statistics based on quadratic entropy and bootstrapping. *J. Statist. Planning and Inference* 43, 1-18.

34. (with R. Boudreau). Graphical representation of blood group data of human populations. *J. Quantitative Anthropology* (special issue) 5, 191-214.

35. (with Rahul Mukerjee). On posterior credible sets based on score statistic. *Statistica Sinica* 4, 781-792.

36. (with Rahul Mukerjee). Comparison of Bartlett-type adjustments for the efficient score statistics. *J. Statist. Planning and Inference* 46, 137-146.

37. (with L.C. Zhao). Strassen's law of iterated logarithms for Lorenz curves. *J. Multivariate Analysis* 54, 239-252.

38. (with L.C. Zhao). Convergence theorems for the cumulative quantile regression functions. *Mathematical Methods of Statistics* 4, 81-91.

39. (with L.C. Zhao). Recent contributions to censored regression models. *Metrika* 42, 203-213.

40. A review of canonical coordinates and an alternative to correspondence analysis using Hellinger distance. *Questio* 19, 23-63.

41. The use of Hellinger distance in graphical displays of contingency table data. In *New Trends in Probability and Statistics* Vol 3, *Multivariate Statistics and Matrices in Statistics,* 143-162, Proc. 5-th Tartu Conference, Estonia.

42. (with T.S. Arthanari). Data analysis and statistical thinking for quality and productivity improvement. *Proceedings of International Conference on Statistical Methods and Statistical Computing for Quality and Productivity*, Vol 1, 14-23, The Korean Statistical Society.

43. (with D.N. Shanbhag). Characterizations based on regression properties: extended versions of recent results. *Sankhyā* A, 57, 167-178.

1.2 M.B. Rao (Senior Research Associate)

1993

44. (with J. Tiefeng, D. Li and XC Wang). Laws of large numbers and moderate deviations for stochastic processes with stationary and independent increments, *Stochastic Processes and their Applications,* 44, 205-219.

45. (with XC Wang and X. Yang). Convergence rates on strong laws of large numbers for arrays of rowwise independent random elements, *Stochastic Analysis and Applications*, 11, 115-132.

1994

46. (with D. Li and XC Wang). On Feller's criterion on the law of the iterated logarithm, *International Journal of Mathematics and Mathematical Sciences,* 17, 323-340.

1995

47. (with D. Li, T. Jiang and X.C. Wang). Complete convergence and almost sure

convergence for weighted sum of random variables. *J. Theoretical Probability* 8, 49-76.

48. (with D. Li and X.C. Wang). Some results on strong limit theorems for (LB)-space valued random variables. *Statistics and Probability Letters* 23, 247-251.

49. (with D. Li and X.C. Wang). On the strong law of large numbers and the law of the logarithm for weighted sums of independent random variables with multidimensional indices. *J. Multivariate Analysis* 52, 181-198.

50. (with X.C. Wang). Convergence in the *r*-th mean and some weak laws of large numbers for random weighted sums of random elements in Banach space. *Northeastern Mathematical Journal* 11, 113-126.

51. (with D. Li and R.J. Tomkins). A strong law for B-valued arrays. *Proc. Am. Math. Society* 123, 3205-3212.

52. (with T. Jiang and X.C. Wang). Large deviations for moving average processes. *Stochastic Processes and Their Applications* 59, 309-320.

1.3 D. Kundu (Senior Research Associate)

1993

53. Estimating the parameters of undamped exponential signals. *Technometrics*, Vol.35, No.2, 215-218.

54. Asymptotic Theory of least squares estimator of a particular non linear regression model. *Statistics and Probability Letters*, Vol.18, No.1, 13-17.

55. (with A. Mitra). Asymptotic Behavior of the least squares estimators of superimposed exponential signals: main results. *P.C. Mahalanobis Birth Centenary Volume*, Edited by A.K. Datta, Published by Indian Statistical Institute, 663-670.

56. (with R. Kundu). Consistent estimates of super imposed exponential signals when some observations are missing: Part 1. *P.C. Mahalanobis Birth Centenary Volume*, edited by A.K. Datta, Published by Indian Statistical Institute, 679-685.

1994

57. (with A. Mitra, N. Mishra and I.D. Dhariyal). Estimating the ration of the smaller and the larger of the two uniform scale parameters. *Journal of Statistical Computation and Simulation*, Vol.50, 197-211.

58. (with N. Kannan). On modified EVLP and ML methods for estimating superimposed

exponential signals. *Signal Processing*, Vol.39, No.3, 223-233.

59. Estimating the parameters of damped exponential signals. *Computational Statistics and Data Analysis*, Vol.18, No.5, 525-536.

60. A modified Prony algorithm for damped or undamped exponential signals. *Sankhyā*, Ser. A. Vol.56, No.3, 524-544.

1995

61. Consistency of the undamped exponential signals on a restricted parameter space. *Communications in Statistics, Theory and Methods*, Ser. A, 24, 241-251.

62. (with R. Kundu). Consistent estimates of superimposed exponential signals when some observations are missing. *Journal of Statist. Planning and Inference* 44, 205-218.

63. (with A. Maitra). Consistent method of estimating the superimposed exponential signals. *Scandinavian Journal of Statistics* 22, 73-82.

64. (with N. Kannan and A. Maitra). Estimating DOA of signals. *Signal Processing and Communications*, Eds. A. Makur and V.U. Reddy, Tata-McGraw Hill, 63-68.

65. Small sample properties of some parametric and nonparametric methods for detection of signals by Monte-Carlo simulation. *IEE Proceedings, Vision Image and Signal Processing* 142, 181-186.

66. (with N. Kannan and A. Mitra). Estimating DOA of signals. *Signal Processing and Communication*, Eds. A. Makur and V.U. Reddy, Tata McGraw Hill, 63-68.

67. (with A. Maitra). Estimating the parameters of damped exponential signals: A non-iterative approach. *Signal Processing* 46, No.3, 363-368.

1.4 Rahul Mukerjee

1993

68. (with D.K. Dey). Frequentist validity of posterior quantiles in the presence of a nuisance parameter: Higher order asymptotics. *Biometrika* 80, 499-505.

69. (with J.K. Ghosh). Frequentist validity of highest posterior density regions in the multiparameter case. *Ann. Inst. Statist. Math.* 45, 293-302.

70. An extension of the conditional likelihood ratio test to the general multiparameter case. *Ann. Inst. Statist. Math.* 45, 759-771.

71. Rao's score test: Recent asymptotic results. In *Handbook of Statistics* 11 (Eds. G.S. Maddala et al)., North Holland, 363-379.

72. (with S. Sengupta). Some optimal sampling designs in the presence of polynomial trends. *Commun.Statist. - Theor. Meth.* 22, 207-218.

73. (with J.K. Ghosh). On priors match posterior and frequentist distribution functions. *Canadian J. Statist.* 21, 89-96.

74. (with S.K. Basu). Inverse sampling for domain estimation in a stratified population. *Austral. J. Statist.* 35, 293-302.

75. (with J.K. Ghosh). Second order minimaxity under squared error and absolute deviation loss. In *Statistics and Probability - A R.R. Bahadur Festschrift* (Eds. J.K. Ghosh et al), Wiley Eastern, New Delhi, 233-243.

76. (with S.K. Basu). Optimal ordering strategy under risk and its nonparametric estimation. *Amer. J. Math. and Management Sc.* 13, 113-122.

77. (with K. Chatterjee). D-optimal saturated main effect plans for $2xs_2xs_3$ factorials. *J. Combinatorics, Information and System Sciences* (Prof. C.R. Rao Dedication Volume) 18, 116-122.

78. (with K. Chatterjee). Search designs for estimating main effects and searching several two -factor interactions in general factorials. *J. Statist. Plann. Inf.* 35, 131-138.

79. (with A. Sengupta). Comparison between the locally most mean power unbiased and Rao's tests in the multiparameter case. *J. Multivariate Anal.* 45, 9-24.

80. (with N. Mukhopadhyay). On the admissibility of some sequential confidence intervals for the negative exponential location parameter. *Sequential Anal.* 12, 289-295.

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81. (with J.K. Ghosh and P.K. Sen). Second order Pitman closeness and Pitman admissibility. *Ann. Statist.* 22, 1133-1141.

82. (with J.K. Ghosh). Adjusted versus conditional likelihood: Power properties and Bartlett-type adjustment. *J. Roy. Statist. Soc.* B 56, 185-188.

83. (with S. Kageyama). On existence of two symbol complete orthogonal arrays. *J. combinatorial Theory* A 66, 176-181.

84. (with A.M. Dean). Construction of regular, single replicate, two-factor designs. *J.*

Statist. Plann. Inf. 38, 89-104.

85. (with C.R. Rao). Tests based on score statistics: Power properties and related results. *Math. Methods of Statist.* 3, 46-61.

86. (with K. Chatterjee). An application of Hadamard matrices for the construction of main effect plus two plans for 2^n factorials. *Utilitas Math.* 45, 213-218.

87. (with J.K. Ghosh). Higher order comparison of tests: A Bayesian route. In *Essays on Probability and Statistics* (in Honor of Prof. A.K. Bhattacharyya, eds. S.P. Mukherjee et al), Presidency College, Calcutta, 92-101.

88. Comparison of tests in their original forms. *Sankhyā* A 56, 118-127.

89. (with K. Chatterjee). A search procedure using polychomotomies for search linear models with positive error variance. *Statistics & Decisions* 12, 91-103.

90. (with S. Sengupta). A-optimal run orders with a linear trend. *Austral. J. Statist.* 36, 115-122.

1995

91. On E-optima; fractions of symmetric and asymmetric factorials. *Statistica Sinica* 5, 515-533.

92. (with C.R. Rao). On posterior credible sets based on the score statistic. *Statistica Sinica* 5, 781-791.

93. (with C.R. Rao). Comparison of Bartlett-type adjustments for the efficient score statistic. *J. Statist. Plann. Inf.* 46, 137-146.

94. (with S. Gupta). A-efficient designs for bioassays. *J. Statist. Plann. Inf.* 48, 247-259.

95. (with C.F.J. Wu). On the existence of saturated and nearly saturated asymmetrical orthogonal arrays. *Ann. Statist.* 23, 2102-2115.

96. (with J.K. Ghosh). On perturbed ellipsoidal and highest posterior density regions with approximate frequentist validity. *J. Roy. Statist. Soc.* B 57, 761-769.

97. (with S. Sengupta). Optimal estimation of a finite multidimensional population total. *J. Statist. Plann. Inf.* 48, 339-346.

98. (with S. Huda and I.H. Khan). On optimal designs with restricted circular string property. *Computational Statist. and Data Anal.* 19, 75-83.

99. (with J.K. Ghosh). Frequentist validity of highest posterior density regions in the presence of nuisance parameters. *Statistics and Decisions* 13, 131-139.

1.5 Damodar Shanbhag

1993

100. (with C.R. Rao). A stability theorem for the integrated Cauchy functional equation. *A memorial Volume for C.G. Khatri*, 175-184.

101. (with S. Kapoor). Some questions in characterization theory. *The Mathematical Scientist*, 18, 127-133.

102. (with B. Fosam and C.R. Rao). comments on some papers involving the integrated Cauchy functional equation. *Statistics and Prob. Letters*, 17, 299-302.

1994

103. (with B. Fosam). Certain characterization of exponential and geometric distributions. *J. Roy. Statist. Soc.* B, 56, 157-160.

104. (with C.R. Rao and T. Sapatinas). The integrated Cauchy functional equation: some comments on recent results. *Adv. Appl. Prob.* 26, 825-829.

1995

105. (with C.R. Rao). Characterizations based on regression properties: extended versions of recent results. *Sankhyā* A 57, 167-178.

Books published during 1993-95

1. (with D.N. Shanbhag). *Choquet-Deny Type Functional Equations with Applications to Stochastic Models*, 1994, John Wiley.

2. (with H. Toutenburg). *Linear Models: Least Squares and Alternatives*, 1995, Springer Verlag.

3. *Statistics and Truth*. Japanese translation, 1993, Spanish and Polish translations, 1994 and German translation, 1995.

Appendix D

LIST OF TECHNICAL REPORTS ISSUED BY THE CENTER FOR MULTIVARIATE ANALYSIS 1993-1995

under the ARO grant: DAAH04-93-G-0030

1993

93-01 C. Radhakrishna Rao and Rahul Mukerjee. COMPARISON OF BARTLETT-TYPE ADJUSTMENTS FOR THE EFFICIENT SCORE STATISTIC, January 1993

Several Bartlett-type adjustments for the efficient score statistic have been recently proposed in the literature. This article compares them, under contiguous alternatives, with reference to the criteria of maximinity and average power.

Submitted: *Journal of Statistical Planning and Inference*

93-02 S. Huda, I.H. Khan and Rahul Mukerjee. ON OPTIMAL DESIGNS WITH RESTRICTED CIRCULAR STRING PROPERTY, January 1993

Exact optimality results on minimal statistical designs with circular string property under restriction are derived. Optimal approximate designs are also obtained.

Submitted: *Computational Statistics and Data Analysis*

93-03 Rahul Mukerjee and S. Sengupta. A-OPTIMAL RUN ORDERS IN THE PRESENCE OF A LINEAR TREND, January 1993

A-optimal run orders have been considered in the presence of a linear trend with emphasis on non-orthogonal situations where no trend-free run order can be A-optimal. Some possibilities for further extension have also been briefly indicated.

Submitted: *Australian Journal of Statistics*

93-04 Robert Fountain. CONCENTRATION AND CLOSENESS, January 1993

Concentration functions provide a graphical display of the behavior of estimators with respect to a given loss function. Several numerical criteria derived from the concentration function are presented, along with their relationships to classical estimation measures. The connections between concentration functions and

measures of closeness are examined, and two new measures of closeness are proposed.

Submitted: *J. Statistical Planning and Inference*

93-05 C. Radhakrishna Rao, L.C. Zhao and Bin Zhou. MAXIMUM LIKELIHOOD ESTIMATION OF TWO-DIMENSIONAL SUPERIMPOSED EXPONENTIAL SIGNALS, January 1993

The problem of maximum likelihood estimation of the parameters (i.i., frequencies, amplitudes, and noise variance) of two-dimensional superimposed exponential signals is considered. In this paper, a polynomial rooting approach is proposed to obtain the estimates of the two-dimensional frequencies. Strong consistency and limiting distribution are established for the estimates of the parameters. Furthermore, the covariance of the limiting distribution attains the Cramer-Rao lower bound.

Accepted: *IEEE Transactions on Signal Processing*

93-06 C.R. Rao and L.C. Zhao. ASYMPTOTIC NORMALITY OF LAD ESTIMATOR IN CENSORED REGRESSION MODELS, January 1993

In this paper we study the asymptotic behavior of the least absolute deviations (LAD) estimates in censored regression (or censored "Tobit") models. Under some mild conditions, the weak consistency and asymptotic normality of these estimates are established.

Published: *Mathematical Methods of Statistics, 2, 228-239, 1993*

93-07 C. Radhakrishna Rao and Rahul Mukerjee. ON POSTERIOR CREDIBLE SETS BASED ON THE SCORE STATISTIC, January 1993

Conditions for approximate frequentist validity of posterior credible sets based on the score statistic have been derived in the multiparameter case. In the process, explicit expressions are given for the posterior quantiles of the score statistic and a posterior Bartlett-type adjustment is suggested for the same.

Submitted: *Statistical Sinica*

93-08 C.R. Rao. J.B.S. HALDANE: A POLYMATH IN THE INDIAN CONTEXT, February 1993

Published: In *Human Population Genetics* (Ed. P.P. Majumdar) Plenum Press, 1-6, 1993

93-09 C.R. Rao. STATISTICS: AN ESSENTIAL TECHNOLOGY IN ENVIRONMENTAL RESEARCH AND MANAGEMENT, February 1993

Statistical methods as developed and used in decision making and scientific research is of recent origin. The logical foundations of statistics are still under discussion and some care is needed in applying the existing methodology and interpreting results.
Some pitfalls in statistical data analysis are discussed and the importance of cross examination of data (or exploratory data analysis) before using specific statistical techniques is emphasized.
Comments are made on the treatment of outliers, choice of stochastic models, use of multivariate techniques and the choice of software (expert systems) in statistical analysis.
The need for developing new methodology with particular relevance to environmental research and policy is pointed out.

Accepted: *J. of Environmental Statistics*

93-10 Steven F. Arnold. GIBBS SAMPLING, February 1993

In this paper, we present an elementary introduction to Gibbs sampling, both multiple path and single path. Gibbs sampling is a Markov sampling scheme which can often be used to approximate posterior distributions in Bayesian models. We indicate how Gibbs sampling can be applied to difficult Bayesian models, hierarchical Bayesian models, Bayesian models with missing data, genetic linkage models and image reconstruction models. In order to keep this paper fairly elementary, derivations are limited to the discrete case. We have also kept the examples as simple as possible. References are given for more general derivations and more complicated examples.

Published: *Handbook of Statistics Vol. 9, Computational Statistics,* (North Holland, Ed. C.R. Rao), 599-626, 1993.

93-11 L.C. Zhao. LEAST ABSOLUTE DEVIATIONS ANALYSIS OF VARIANCE IN CENSORED REGRESSION MODELS, February 1993

For testing a linear hypothesis in a censored regression (or censored "Tobit") model, three test criteria based on least absolute deviations estimates of parameters are proposed and their limiting Chi-square distributions are established. Some consistent estimates of nuisance parameters are also obtained for use in computing the test statistics.

Submitted: *Sankhyā*

93-12 C.R. Rao. UNCERTAINTY, STATISTICS AND CREATION OF NEW KNOWLEDGE, February 1993

To appear in: *Contemporary Statistical Thought*

93-13 Zhijung Liu. LEAST WEIGHTED ABSOLUTE DEVIATIONS ESTIMATION IN LINEAR MODELS, March 1993

The least weighted absolute deviations (LWAD) estimate is introduced in this paper to improve the robust performance of the well studied least absolute deviations (LAD) estimate. Under mild conditions, an asymptotic theory of the LWAD estimate is established that includes consistency, asymptotic normality, and Bahadur-Kiefer type representation of the LWAD estimate in linear models. In addition, test statistics for linear hypotheses are given and their asymptotic distributions are derived.

Submitted: *J. Statistical Planning and Inference*

93-14 Debasis Kundu. ASYMPTOTIC THEORY OF LEAST SQUARES ESTIMATOR OF A PARTICULAR NONLINEAR REGRESSION MODEL, March 1993

The consistency and asymptotic normality of the least squares estimator are derived for a particular non-linear regression model, which does not satisfy the standard sufficient conditions of Jennrich (1969) or Wu (1981), under the assumption of normal error.

Published: *Statistics and Probability Letters,* 18, 13-17, 1993

93-15 V.K. Srivastava and H. Toutenburg. APPLICATION OF STEIN-TYPE ESTIMATION IN COMBINING REGRESSION ESTIMATES FROM REPLICATED EXPERIMENTS, March 1993

This paper considers the application of Stein-type estimation procedure for the coefficients in a linear regression model when data are available from replicated experiments. Two families of estimators characterized by a single scalar are proposed and their large sample asymptotic properties are derived. These are utilized for comparing the performances of the two estimators along with the conventional estimator and conditions for the superiority of the estimator over the other are deduced.

Submitted: *Statistical Papers*

93-16 C. Radhakrishna Rao. STATISTICS MUST HAVE A PURPOSE: THE MAHALANOBIS DICTUM, April 1993

Prasanta Chandra Mahalanobis is one of the pioneers who laid the foundations of statistics as a separate discipline. His main contributions to statistical theory and applications are multivariate methods in taxonomy (Mahalanobis distance), optimum design of large scale sample surveys, and use of econometric models in planning. He was the founder of the Indian Statistical Institute (ISI), which is internationality recognized for its educational and interdisciplinary research programs. He was also responsible for organizing and developing the Indian statistical systems which is considered to be one of the best in the world. He considered statistics as a new technology of the present century which can be applied to any field of human endeavor, and suggested that statistics should be collected in a planned manner keeping the purpose in view.

Published: *Proceedings of the International Statistical Institute,* 49-th session, Firenze, IN01, 21-36, 1993

93-17 C. Radhakrishna Rao and Paula M. Caligiuri. ON SCALING OF ORDINAL CATEGORICAL DATA, April 1993

This paper discusses the analysis of data when responses are in the form of categories in a multiway contingency table. The methodology is based on quantifying or scaling the categories of each attribute and applying the usual multivariate techniques developed for continuous variables. In the case of ordinal categories, the scales are obtained to match the natural order. This enables a meaningful interpretation of results based on estimated scales.

Published: In *Multivariate Analysis: Future Directions 2* (North Holland, Eds. C.M. Cuadras and C.R. Rao), 97-112, 1993

93-18 C.R. Rao and L.C. Zhao. A LIMITING DISTRIBUTION THEOREM FOR TESTING EXPONENTIALITY BASED ON SAMPLE ENTROPY, June 1993

Under some mild conditions, we establish the limiting distribution of a test statistic proposed by Ebrahimi and Habibullah (1992) for testing exponentiality based on sample entropy.

Submitted: *Le Cam Festschrift Volume*

93-19 H. Toutenburg and V.K. Srivastava. ESTIMATION OF REGRESSION COEFFICIENTS SUBJECT TO INTERVAL CONSTRAINTS, May 1993

In this article, we have postulated a linear regression model which is subject to a set of interval constraints on its regression coefficients. Recognizing that it is generally difficult to derive explicit expressions for the estimators of regression coefficients and to study their properties, two alternative formulations of

constraints (one in the form of ellipsoidal restrictions and the other in the form of linear stochastic restrictions) are considered and efficiency properties of the resulting estimators are studied employing the small disturbance asymptotic theory.

Submitted:

93-20 Deli Li and M. Bhaskara Rao. A NOTE OF THE LAW OF THE ITERATED LOGARITHM FOR WEIGHTED SUMS OF INDEPENDENT IDENTICALLY DISTRIBUTED RANDOM VARIABLES, July 1993

A triangular array of real numbers and a sequence of independent identically distributed random variables are provided for which the law of the iterated logarithm fails.

Submitted:

93-21 Kathy Kraft and M. Bhaskara Rao. EFFECTS OF RANDOM AND DETERMINISTIC INSPECTION ON THE ASYMPTOTIC PROPERTIES OF ESTIMATORS IN SURVIVAL ANALYSIS, July 1993

In modeling the lifetimes of individuals from a target population, it is virtually impossible to monitor the individuals in the sample continuously in a variety of situations. In this paper, we look at two types of inspection policies: a deterministic inspection policy, in which all individuals in the sample are observed at predetermined times and a random inspection policy, in which all individuals in the sample are observed at times determined by a stochastic process. We examine the effect of the inspection policies on the asymptotic properties of the maximum likelihood estimators of the parameters of a simple random censorship model.

Submitted:

93-22 Manzoor Hussain and M. Bhaskara Rao. A NOTE ON E-OPTIMAL MINIMAL BLOCK DESIGNS UNDER MIXED EFFECTS MODEL, July 1993

Mukerjee, Shah and Sinha (1992) identified the E-optimal design under mixed effects model from among the class of all minimal block designs which are connected under fixed effects model. In this note, we show that the same design is E-optimal under mixed effects model from among the class of all minimal block designs. But this design does not enjoy the uniqueness property any longer.

Submitted: *Metrika*

93-23 V.V. Bapeswara Rao and M. Bhaskara Rao. A GENERALIZATION OF THE JERUSALEM TICKET PROBLEM, July 1993

Suppose $a_1, a_2, \ldots, a_n$ tickets are required for n performances $A_1, A_2, \ldots, A_n$, respectively. Suppose a free ticket is offered for every k tickets bought for distinct performances. This note establishes a formula for the maximum number of free tickets utilizable.

Submitted: *Journal of Algorithms*

93-24 Z.D. Bai, K. Subramanyam and L.C. Zhao. DETERMINATION OF THE ORDER OF ARIMA PROCESSES, August 1993

In this paper, using a general theoretic criterion, a new method to estimate the order of autoregressive integrated moving average (ARIMA) model is proposed. This procedure yields a strongly consistent estimate of the orders of ARIMA model.

Submitted: *Sankhyā*

93-25 Xin W. Jia and M. Bhaskara Rao. STATISTICAL MODELING OF JOINT ACTION OF DRUGS: THE CASE OF BIVARIATE WEIBULL DISTRIBUTION, August 1993

A bivariate Weibull distribution is proposed to model joint action of two drugs. The parameters are estimated through one step using likelihood principle. Biological independent joint action of two drugs is studied under the proposed bivariate Weibull family. Some examples are presented for illustration.

Submitted: *Biometrics*

93-26 C.R. Rao and L.C. Zhao. STRASSEN'S LAW OF THE ITERATED LOGARITHM FOR THE LORENZ CURVES, September 1993

Under some mild conditions we establish the Strassen's law of the iterated logarithm for the Lorenz curves.

Submitted: *J. Multivariate Analysis*

93-27 C. Radhakrishna Rao, L.C. Zhao and B. Zhou. A NOVEL ALGORITHM FOR 2-DIMENSIONAL FREQUENCY ESTIMATION, October 1993

In this paper, an algorithm for 2-D frequency estimation is proposed. This algorithm consists of two parts. First, two sets of 1-D distinct frequencies are

estimated separately. Then, the estimates of the 2-D frequencies are obtained by pairing the components in the two sets of estimated 1-D frequencies. The correct pairing is accomplished based on the fact that a vector in the signal subspace is orthogonal to the noise subspace of a 2-D signal. The capability of determining the number of signals is inherited in the proposed algorithm. Computer simulation results are provided to demonstrate the efficacy of the algorithm.

To appear: *Proceedings of the 27-th Asilomar conference on signals, systems and computers*

93-28 C.R. Rao and L.C. Zhao. CONVERGENCE THEOREMS FOR EMPIRICAL CUMULATIVE QUANTILE REGRESSION FUNCTION, November 1993

Let (X,Y) be a bivariate random variable and $F(x)$ be the marginal distribution function of X. We define $r(u)=E(Y \mid F(X)=u)$ as the quantile regression (QR) function of Y on X. The cumulative QR function $M(u)$ is defined as the integral of $r(\cdot\ )$ over the range $[0,u]$. A standardized form of $M(u)$ is $L(u)=[M(1)]^{-1}M(u)$. [It is seen that in the special case of $Y=X$, $r(u)$ is the quantile function, and $L(u)$ is the Lorenz concentration curve.] In the general case, we estimate $M(u)$ and $L(u)$ from a given sample of size n by $M_n(u)$ and $L_n(n)$ and study their convergence properties as $n\to\infty$. We also consider test statistics for comparing the CQR (cumulative QR) functions of two populations.

Submitted: *Mathematical Methods of Statistics*

93-29 Abdulghani A. Alharbi, D.N. Shanbhag. GENERAL CHARACTERIZATION THEOREMS BASED ON VERSIONS ON THE CHERNOFF INEQUALITY AND THE COX REPRESENTATION, December 1993

In this communication, we extend characterization theorems for distributions based on versions of the Chernoff inequality and relate these to Cox's representation for a survivor function in terms of the hazard measure, as presented by Kotz and Shanbhag (1980). (The original version of this appeared in Cox (1972).) Some corollaries of the results, are also presented.

Submitted: *J. Statist. Planning and Inference*

93-30 C.R. Rao, T. Sapatinas and D.N. Shanbhag. THE INTEGRATED CAUCHY FUNCTIONAL EQUATION: SOME COMMENTS ON RECENT PAPERS, December 1993

We make some comments on recent papers involving the integrated Cauchy functional equation or specialized versions of it, and reveal in particular that these give an inaccurate picture of the current state on the literature on the topic.

Accepted: *J. Applied Probability*

1994

94-01 C.R. Rao and L.C. Zhao. LAW OF THE ITERATED LOGARITHM FOR EMPIRICAL CUMULATIVE QUANTILE REGRESSION FUNCTIONS.

Under some mild conditions we establish the Strassen's law of the iterated logarithm for the empirical cumulative quantile regression function.

Submitted:

94-02 Dongsheng Tu and Alan J. Gross. ON THE ACCURACY AND BARTLETT TYPE CORRECTION FOR THE SUBJECT-YEARS METHOD IN COMPARING SURVIVAL DATA TO A STANDARD POPULATION.

In medical follow up studies, the subject years method is often used to compare the mortality experience of the population being studied to that of a known population. We develop in this paper an asymptotic expansion for the null distribution of the test statistic derived from this method. Based on this expansion, the accuracy of the subject years method is discussed and a modified test statistic with a higher order accuracy is proposed. The actual type I errors and powers of the new test are compared with that of the original test and some other related tests by Monte-Carlo simulations. The proposed test is also applied to analyze the survival time (mortality) data for psychiatric patients with schizophrenia, mania and depression.

Submitted:

94-03 Helge Toutenburg and Sung H. Park. MISSING VALUES IN REGRESSION: MIXED AND WEIGHTED MIXED ESTIMATION.

The paper is related to regression models with a nonstochastic regressor matrix where some elements are nonobserved. Besides the common missing values procedures there are discussed some aspects of the maximum likelihood estimator in the light of mixed estimation. The mixed regression framework is the central method of this paper. Replacing missing values leads to biased mixed estimators which are compared with the classical least squares estimator according to strong and weaker MSE-criteria in the sense of Wallace (1970). Weighting the replaced submodel results in a weighted mixed regression estimator whose operational form os a two-stage estimator.

Submitted:

94-04 D. Kundu and R. Kundu. CONSISTENT ESTIMATES OF SUPER IMPOSED EXPONENTIAL SIGNALS WHEN SOME OBSERVATIONS ARE MISSING.

Methods are proposed for estimating the parameters of undamped exponential signals when observations are missing. Some consistency results have been established. The finite sample behavior of the proposed methods have been studied by Monte Carlo simulation.

Submitted: *J. Statistical Planning and Inference.*

94-05 C.R. Rao. A REVIEW OF CANONICAL COORDINATES AND AN ALTERNATIVE TO CORRESPONDENCE ANALYSIS.

In this paper, a general theory of canonical coordinates is developed for reduction of dimensionality in multivariate data, assessing the loss of information and plotting higher dimensional data in two or three dimensions for visual displays. The theory is applied to data in two way tables with variables in one category and samples (individual or populations) in the other. Two types of data are considered, one with continuous measurements on the variables and another with frequencies of attributes. An alternative to the usual correspondence analysis of contingency tables based on Hellinger rather that the chisquare distance is suggested. The new method has some attractive features and does not suffer from some inherent drawbacks resulting from the use of the chisquare distance and variable sample sizes for the populations in the correspondence analysis. The technique of biplots where the populations and the variables are represented on the same chart is discussed.

Submitted:

94-06 C.R. Rao and L.C. Zhao. RECENT CONTRIBUTIONS TO CENSORED REGRESSION MODELS.

A briefly survey of estimation of parameters in a censored regression model (known as the Tobit model) and some details of the properties of LAD (least absolute deviation) estimates and tests of significance of linear hypotheses are given.

Submitted:

94-07 C.R. Rao. THE USE OF HELLINGER DISTANCE IN GRAPHICAL DISPLAYS OF CONTINGENCY TABLE DATA.

A general theory is developed for representing population profiles characterized by multiple measurements in a low dimensional Euclidean space. The basic input

to the problem is a measure of distance between two population profiles. A well known method for representing row or column profiles in a contingency table using a chisquare type distance between profiles if the correspondence analysis. It is suggested that a similar analysis based on Hellinger distance between profiles has some advantages and is better suited for studying the configuration of profiles. Some examples are given to show the differences between the two analyses.

An asymmetric biplot technique which is useful in interpreting differences in row (column) profiles in terms of column (row) categories is developed.

Paper presented at the Fifth Tartu Conference on Multivariate Statistics.

94-08 Deli Li, M. Bhaskara Rao and R.J. Tomkins. THE STRONG LAW OF LARGE NUMBERS AND THE LAW OF THE ITERATED LOGARITHM FOR LINEAR COMBINATIONS OF ORDER STATISTICS.

L-statistics play an important role in a variety of areas in Statistics and Probability. Kolmogorov type Strong Laws of Large Numbers are well known for these statistics. In this article, Marcinkiewicz-Zygmund type Strong Laws of Large Numbers and bounded Hartman-Wintner type Law of the Iterated Logarithm are established for linear combinations of order statistics with smooth score functions under a minimal moment condition, The Law of the Logarithm for weighted sums of order statistics is also established.

Submitted: *Stochastic Processes and Their Applications.*

94-09 Deli Li, M. Bhaskara Rao and R.J. Tomkins. A STRONG LAW FOR B-VALUED ARRAYS.

Let $(B, \|\cdot\|)$ be a real separable Banach space and $\{X_{n,k}; \geq 1, 1 \leq k \leq n\}$ a triangular array of iid B-valued random variables. Set $S(n) = \sum_{k=1}^{n} X_{n,k}, n \geq 1$ and $\text{Log } t = \log \max \{e,t\}, t \in \Re$. In this paper, we characterize the limit behavior of $S(n)/\sqrt{2n\text{Log } n}, n \geq 1$. As an application of our result, we resolve an open problem posed by Hu and Weber (1992). The case of row-wise independent arrays is also dealt with.

Submitted: *Proc. Amer. Math. Soc.*

94-10 Xiangchen Wang and M. Bhaskara Rao. MODERATE DEVIATIONS FOR RANDOM GEOMETRIC SERIES WITH APPLICATIONS TO THE LAW OF THE ITERATED LOGARITHM.

Let $\xi(\beta) = \sum_{n \geq 0} \beta^n \epsilon_n$, $\beta \in (0,1)$, be a geometric random series, where $\{\epsilon_n; n \geq 0\}$ are i.i.d. random variables with mean 0 and variance σ^2. We compute moderate

deviations of the series under the assumption that E exp $(t\epsilon_0) < \infty$ exists in a neighborhood of the origin. The results are used to prove the law of the iterated logarithm for the series, when $\{\epsilon_n; n \geq 0\}$ is an i.i.d. Bernoulli sequence.

Submitted: *Statistics and Probability Letters.*

94-11 Min Deng. THE GENERAL EXPLICIT FORMULA FOR THE PROBABILITY OF RUIN.

This paper focuses on the important aspect of Risk Theorem - the probability function of ruin. The general explicit formula for the probability function of ruin has been derived in the following two cases.

1. Two parameters case: i.e. the probability density function of claim amount is given by $p(x)=pae^{-ax}+qbe^{-bx}$,
2. Three parameters case: i.e. the probability density function of claim amount is given by $p(x)=p_1a_1e^{-a}1^x+p_2a_2e^{-a}2^x+p_3a_3e^{-a}3^x$.

The theoretic discussion involved very complex computation. Most of the computations have been done by "Mathematica" - a mathematical software. We also discuss the results by Tsao and point out that his results are the special cases of our results.

Submitted:

94-12 Min Deng. A GENERALIZED RISK MODEL AND ITS PROPERTIES.

In this paper, we consider a generalized risk model, where we assume that the occurrence of the claims is according to a mixed Poisson distribution instead of a Poisson distribution, and discuss the following questions.

1. The recursive formula.
2. The probability of ruin.
3. The upper bound of the probability of ruin.
4. The probability of ruin when the initial surplus is zero.
5. The probability of ruin when the claim costs are exponentially distributed with parameter β.

Submitted:

94-13 C.R. Rao and D.N. Shanbhag. A NOTE ON A CHARACTERISTIC PROPERTY BASED ON ORDER STATISTICS.

It is shown that the extended version of the Puri-Rubin result given recently by Stadje (1994) is neither new nor the most general available in the literature.

Submitted: *Amer. Math. Soc.*

94-14 Debasis Kundu. CONSISTENCY OF THE UNDAMPED EXPONENTIAL SIGNAL MODEL ON A RESTRICTED PARAMETER SPACE.

The main aim of this note is to prove the consistency of the least norm squares estimates of an undamped exponential signal model on a restricted parameter space and to compare different methods available in the literature to compute the least norm squares estimates.

Submitted:

94-15 Amit Mitra, Debasis Kundu, Neeraj Misra and I.D. Dhariyal. ESTIMATING THE RATIO OF THE SMALLER AND THE LARGER OF TWO UNIFORM SCALE PARAMETERS.

Independent random samples are drawn from two uniformly distributed populations with unknown scale parameters. The problem is to estimate the ratio of the minimum and the maximum of the two scale parameters. It is important because, in ranking and selection problem, the true probability of correct selection {P(CS)} is a function of this ratio. In this paper several estimators are proposed and their performances are compared empirically.

Submitted:

94-16 Debasis Kundu, Amit Mitra. CONSISTENT METHOD OF ESTIMATING SUPERIMPOSED EXPONENTIAL SIGNALS.

A consistent non-iterative method is proposed for estimating the parameters of undamped exponential signals when the parameters are complex valued. Such data arise in several areas of applications, including telecommunication, seismic signal processing and computer assisted medical diagnostics. Among the non-iterative methods, the best known is the modified Forward Backward Linear Prediction method of Tufts and Kumaresan, which is not consistent. It is observed that the proposed method is consistent and provides accurate frequency estimates at lower signal to noise ratio than the currently existing non-iterative techniques.

Submitted:

94-17 Amit Mitra and Debasis Kundu. ASYMPTOTIC BEHAVIOR OF LEAST SQUARES ESTIMATES OF SUPERIMPOSED EXPONENTIAL SIGNALS.

Recently Rao and Zhao (1993) obtained the asymptotic properties of the least squares estimates of the superimposed exponential signals model under the assumption of the complex normal distribution of the error term. In this paper we extend the result to a larger class of distributions. The two approaches are also

totally different.

Submitted:

94-18 C.R. Rao. SOME STATISTICAL PROBLEMS ASSOCIATED WITH SIGNAL PROCESSING.

The object of this communication is to report on some recent results on statistical analysis and inference on signals in one and two dimensional superimposed exponential signal models, and to mention some outstanding problems in this area.

Submitted: *Proc. 28th Asilomar Conference on Signals, Systems and Computers.*

1995

95-01 E.B. Fosam and D.N. Shanbhag. VARIANTS OF THE CHOQUET-DENY THEOREM WITH APPLICATIONS.

A characterization of the exponential distribution based on a relevation-type equation and its discrete version are extended to the case of multidimensional spaces via variants of the Choquet-Deny Theorem. Comments on some recent results in the literature are made.

Submitted: *J. Statistical Planning and Inference*

95-02 D. Kundu. SMALL SAMPLE PROPERTIES OF SOME PARAMETRIC AND NONPARAMETRIC METHODS FOR DETECTION OF SIGNALS BY MONTE CARLO SIMULATION.

We consider the estimation of the number of signals in a multichannel time series model. We use different information Theoretic Criteria and some nonparametric methods to detect the number of signals and compare its small sample properties by Monte Carlo Simulation study.

Submitted: *Proceedings IEE - Vision, Image and Signal Processing*

95-03 Z.D. Bai, C.R. Rao and Y.H. Wu. M-ESTIMATION OF MULTIVARIATE LINEAR REGRESSION BY MINIMIZING THE DIFFERENCE OF TWO CONVEX FUNCTIONS.

M-estimates of the parameters in a linear model are obtained by minimizing the sum of functions of residuals. The function called the "discrepancy function", denoted by p, is chosen to provide robust estimates. In this paper we suggest the

choice, $p=p_1-p_2$, where p_1 and p_2 are convex functions. By choosing p_1 and p_2 suitably, we can generate a wide variety of discrepancy functions which can be used in practice. There is an added advantage that the conditions placed on p_1 and p_2 and hence on p are much milder than those considered in the literature on M-estimation.

Submitted: *Handbook of Statistics: Robust Inference, Vol. 15*

95-04 C.R. Rao and R. Mukerjee. COMPARISON OF LR, SCORE AND WALD TESTS IN A NON-IID SETTING.

Considering a large class of tests, we study higher order power in a possibly non-iid setup. Optimum properties for the likelihood ratio and score tests are exhibited under the criteria of second-order local maximinity and third-order local average power respectively. The issue of stringency with regard to third order average power has been addressed. We also compare the power properties of various Bartlett-type adjustments for the tests.

Submitted: *J. of Multivariate Analysis.*

95-05 C.R. Rao and D.N. Shanbhag. A CONJECTURE OF DUFOUR ON A CHARACTERIZATION OF THE EXPONENTIAL DISTRIBUTIONS. Distributions.

Dufour gave a conjecture on a characterization of the exponential distributions based on type 2 right-censored samples. The conjecture was shown to be true recently by Leslie and van Eeden (1993) in the case when the number of censored observations is no larger than (1/3)n-1, where $n(\geq 3)$ is the sample size. We now show that the conjecture is valid when the number of censored observations is less than or equal to $\max\{0,n-5\}$, and also give a general identifiability result subsuming the Leslie - van Eeden result. The results have implications for testing fit of censored data.

Submitted: *Festschrift of S. Kotz.*

95-06 C.R. Rao and T.S. Arthanari. DATA ANALYSIS AND STATISTICAL THINKING FOR QUALITY AND PRODUCTIVITY IMPROVEMENT.

Experience in implementing quality has identified a core body of useful statistical concepts, methods and tools. The integrating concept is statistical thinking which focuses on generation of data, extraction of relevant from data and utilization of information for optimal decision making. The tools and methods helpful in statistical thinking include graphical techniques, design of experiments, sample surveys, statistical process control and Taguchi concepts and methods. The

expanding use of statistical thinking will not only improve system performance, but will also result in better management of an organization, more delighted customers and more effective leadership. Some examples and concept diagrams will be given illustrating th importance of statistics as a discipline in the optimum utilization of human and material resources for improving quality and productivity.

Submitted: *Proceedings of the International Conference on Statistical Methods and Statistical Computing for Quality and Productivity Improvement (ICSQP'95).*

95-07 T. Jiang, M.B. Rao and X. Wang. MODERATE DEVIATIONS FOR RENEWAL PROCESSES.

Let $X_n, n \geq 1$ be a sequence of independent and identically distributed random variables with $EX_1 = \mu > 0$. Define the sequence $S_n = X_1 + X_2 + \ldots + X_n, n \geq 1$ of partial summands and the renewal process $N(t) = \inf\{n \geq 1; S_n > t\}, t > 0$. In this paper, we give first a strong law of large numbers and then compute moderate deviations for the process $(N(t) - (t/\mu))/(t^{1/2}\varphi(t)), t > 0$ as $t \to \infty$, where the function $\varphi(.)$ satisfies some regularity conditions. As an application of the above results, we obtain a moderate deviation result for the randomly indexed process $(S_{N(t)} - N(t)\mu/t^{1/p}, t > 0$ with $p \in (1,2)$. Using a result of Jiang (1994) on large deviations for renewal processes, we obtain a large deviation result for the process $\{S_{N(t)}/N(t), t > 0\}$. An example is provided as an illustration.

Submitted: *Annals of Applied Probability.*

95-08 T.S. Arthanari. GAME THEORY AND QUALITY ENGINEERING.

Dr. Genichi Taguchi has brought in a revolution in the thinking of research and development in industries to design new products and develop new processes. Since then enormous interest has been shown by the practitioners in his methods known as Taguchi Methods. There has been controversy concerning these methods from a statistical standpoint. G.E.P.Box and others have pointed out that some of these techniques are inefficient and unnecessarily complicated. This is particularly true of S/N ratios and statistical analysis based on them. S/N ratios play a crucial role in implementing Taguchi's concept of robustness of product and process design, in Quality Engineering. In this paper, we give a two-person zero-sum game formulation of the S/N ratio analysis to select a robust design. This approach has the advantage of presenting to the designer both the mean and the standard deviation corresponding to a design setting instead of a single number such as S/N ratio. We present here some new results and illustrate out method with examples.

Submitted: *Proceedings of the Conference of Game Theory & Quality*

Engineering.

95-09 X. Jia and M.B. Rao. MODELING RESPONSE DATA OF JOINT ACTION OF TWO DRUGS APPLIED ALONE AND TOGETHER.

In modeling joint action of two drugs using dose response data, following the classical theory, two components are involved: (1) a model for the joint distribution of individual tolerances for the drugs and (2) a model for the response region of dies of the drugs for given individual tolerances of the drugs. In the literature a lot of attention is bestowed on the second component of the modeling problem. In this paper, we introduce a new approach of modeling response data of joint action of two drugs, which includes a new model for the joint distribution of tolerances for two drugs and a new parametric model for the response region. The new models are then fitted to some response data. Contrary to be usual multi-step procedure used in fitting models in the literature, we use a single step maximum likelihood procedure to estimate all the parameters involved in both the joint tolerance distribution and the response region. The main hypothesis of interest is to test if the given response data of two drugs when applied singly and together can be modeled following the new approach.

Submitted: *Biometrics.*

95-10 X. Jia and M.B. Rao. AN ALTERNATIVE MODEL FOR JOINT ACTION OF TWO DRUGS.

A new model for joint action of drugs is introduced which includes several existing models as special cases. Some examples are presented to illustrate the use of the new model.

Submitted: *Proc. of Amer. Stat. Associ. Biopharmaceutical section.*

95-11 X. Jia and M.B. Rao. A NEW MODELING APPROACH FOR JOINT ACTION OF TWO DRUGS.

In the literature of quintal response analysis for joint action of two drugs, modeling response region and modeling the joint tolerance distribution have been two endeavors. Following this line, we develop a new modeling approach to provide a method used in modeling joint actions. We propose a new response region which is shown to cover some types of joint action of two drugs, and a bivariate Weibull model (BVWD) for the tolerances of two drugs. The basic compromise of the new approach is its computational ease using the maximum likelihood (ML) methodology; and this computational advantage is mainly due to easy-to-manipulate property of the bivariate Weibull model, and is partly due to the simple structure of our response region. Because of the constantly growing

popularity of univariate Weibull distributions in modeling quanta response data for a single drug, our bivariate Weibull model points out a reasonable choice for joint tolerance distributions of two drugs and indeed, by fitting real data from the literature, can give a good model fitting and testing for joint action.

Submitted: *Proc. Amer. Stat. Associ. Biopharmaceutical section.*

95-12 R. Nishii, S. Kusanobu and S. Tanaka. SPATIAL RESOLUTION ENHANCEMENT OF LOW RESOLUTION THERMAL INFRARED IMAGE BASED ON OTHER HIGH RESOLUTION IMAGES.

The Landsat TM sensor for thermal infrared measurements, Band 6, has lower resolution than those of other six Bands. In this article we propose a statistical approach to enhance the resolution of low resolution thermal infrared image by using other band images. We employ a multivariate normal distribution for the joint distribution of 7 band values. The values of Band 6 are predicted by the conditional expectations.

Submitted: *Proceedings of the Statistics Day.*

95-13 C.R. Rao and M.B. Rao. TWENTIETH CENTURY STATISTICS.

The article traces the development of statistics as a separate discipline of study during the present century. Statistical reasoning is based on inductive logic of generalizing from the particular to the general (from a sample to the population). Since such a process involved some amount of uncertainty, it was not clear how to make judgements or take optimal decisions. The breakthrough occurred in the beginning of the present century by recognizing that by quantifying uncertainty, rules can be formulated for taking decisions subject to minimal risk. This opened the flood gates to the application of statistics in all fields of human endeavor, ranging from natural and social sciences, engineering and technology, management and economic affairs to arts, literature and legal problems.

Submitted: *Encyclopedia Italiana.*

95-14 C.R. Rao. SEVEN INEQUALITIES IN STATISTICAL ESTIMATION THEORY.

When I was learning statistics as a student, the only serious literature available on statistical theory and practice was Fisher's book *Statistical Methods for Research Workers* and his numerous papers on estimation and small sample exact distributions of some test statistics. Fisher was laying the logical foundations of statistics as a separate discipline. He introduced numerous ideas and analytical tools which we still use. Fisher was criticized for lack of rigor in mathematical

rendering of his ideas and in proving his results, and for making general statements without specifying the underlying assumptions (Savage, 1976). My understanding of statistics is based on reading each of Fisher's results n times depending on the complexity of the problem involved. I am still reading some of his papers. In this process, I developed some inequalities to interpret and clarify Fisher's ideas. I shall describe them to you with some historical notes.

Submitted: *Student.*

95-15 E.B. Fosam and D.N. Shanbhag. AN EXTENDED LAHA-LUKACS CHARACTERIZATION RESULT BASED ON REGRESSION PROPERTY.

This note extends the Laha-Lukacs characterization result based on a regression property subsuming, the Letac-Mora characterization of the natural exponential families of distributions with variances as cubic functions of means. It also makes relevant comments on the existing literature and points out, amongst other things, that certain known characterization results on power series distributions are corollaries to analogous results on exponential families.

Submitted: *Statistical Planning and Inference.*

95-16 D. Kundu and A. Mitra. ASYMPTOTIC PROPERTIES OF THE LEAST SQUARES OF 2-D EXPONENTIAL SIGNALS.

Recently Rao et.al. (1994) established the strong consistency and asymptotic normality of the maximum likelihood of the 2-D superimposed exponential signal model under the assumption of normality of the error random variables. In this paper we investigate the theoretical properties of the least squares estimates of the same model under the assumption of general error distribution. The strong consistency and asymptotic distribution of the least squares estimates have been obtained. Further extension to the multidimensional case has been proposed.

Submitted: *Multidimensional Systems and Signal Processing.*

95-17 D. Kundu and A. Mitra. ESTIMATING THE PARAMETERS OF EXPONENTIALLY DAMPED/UNDAMPED SINUSOIDS IN NOISE: A NON-ITERATIVE APPROACH.

A non-iterative method is proposed for estimating the parameters of damped/undamped exponential signals in noise. In this paper we present a new method based on the decomposition of the noise space of an extended order model. It is observed that the proposed method provides better estimates even at lower signal to noise ratio than currently existing non-iterative techniques in terms of lower mean squared errors.

Submitted: *Signal Processing.*

95-18 N. Kannan. ESTIMATING DOA OF SIGNALS.

High resolution parameter estimation is a significant problem in signal processing applications. Such applications include direction of arrival (DOA) estimation for narrow band signals emitted by multiple sources and received by sensor arrays. It is well known that among the several methods, the centra symmetric TLS-ESPRIT and MUSIC outperform the other methods. In this paper, we propose some new eigenspace decomposition methods and modify some of the existing methods in estimating DOA of signals using both the data vector and its conjugate. It is observed that the new methods work better than TLS-ESPRIT and MUSIC in terms of lower mean squared errors at least for small SNR and computational costs of some of the methods are less than that of TLS-ESPRIT. The strong consistency of the estimators has been established. Simulation results that illustrate the performance of the different methods are presented.

Signal Proc. and Comm., eds. A. Makur and V.U. Reddy, Tata McGrow Hill, N. Delhi, 63-68.

95-19 Z.D. Bai, D. Kundu, A. Mitra. A NOTE OF THE CONSISTENCY OF MULTIDIMENSIONAL EXPONENTIAL SIGNALS.

In this note we present a result in probability which does not exist as it is in the literature of probability theory. We show that this result can be used to prove the consistency of the least squares estimates of the multidimensional exponential signals model under weaker assumptions than that exists in the literature.

Submitted: *Sankhyā* A.

95-20 D. Kundu and A. Mitra. ASYMPTOTIC THEORY OF LEAST SQUARE ESTIMATOR OF A NONLINEAR TIME SERIES REGRESSION MODEL.

The consistency and asymptotic normality of the least squares estimator are derived of a particular non-linear time series model. It does not satisfy the standard sufficient conditions of Jennrich (1969) or Wu (1981). The errors are assumed to be independently and identically distributed random variables each with mean zero and finite variance. Walker (1971) considered the same model and obtained the asymptotic properties of an approximate least squares estimator. It is observed that the least squares estimator and the approximate least squares estimator are asymptotically equal. Some simulations have been performed to compare the two for small samples.

Submitted: *Comm. in Stat.* A.

95-21 B. Chandra and B. Krishna Murthy. SYNTACTIC ANALYSIS OF INDIAN LANGUAGES USING RBF & BP NETWORKS.

In this paper, the use of neuroco,putting for syntactic analysis of sentences in Indian Languages is presented. A heterogenous combination of neural networks namely: Radial Basis Function and Backpropagation networks are cascaded to form a multi stage feedforward network system. The system has four stages category net, Morph net, phrase net and Syntactic net. The output of each stage is given as input to the next stage. Telugu language has been taken to illustrate with examples and to demonstrate the applicability of Neurocomputing technology for Natural Language Processing.

Submitted: *Neural Networks J.*, 1995.

95-22 D. Kundu. MODIFIED MUSIC ALGORITHM FOR ESTIMATING DOA OF SIGNALS.

High resolution signal parameter estimation is a significant problem in many signal processing applications. Such applications include direction of arrival (DOA) estimation for narrow band signals emitted by multiple sources and received by sensor arrays. It is well known that MUSIC algorithm out performs any other method existing in the literature. In this article a modified MUSIC algorithm is proposed using the conjugate data. Strong consistency of the modified method is established. It is observed that the modified MUSIC works significantly better than the ordinary MUSIC at different SNR in terms of the mean squared error.

Submitted: *Signal Processing.*

95-23 C.R. Rao. UNCERTAINTY, STATISTICS AND CREATION OF NEW KNOWLEDGE.

How do we take decisions under uncertainty? How do we generalize from particular observed data to discover new phenomena or to advance natural knowledge? Is the process involved an art, technology of science? What are its philosophical and logical implications? How does statistics, the newly developed discipline of the present century to study uncertainty, help in this process?

Submitted: *Chance.*

95-24 C.R. Rao. EMPIRICAL BAYES AND HIERARCHICAL BAYES PROCEDURES IN SIMULTANEOUS ESTIMATION OF PARAMETERS.

We consider a (p+q)-vector random variable (θ,y), where θ is p-dimensional and

y is q-dimensional. Let $(\theta_1,y_1),\ldots,(\theta_n,y_n)$ be k independent successive samples on (θ,y) but with only y_i values observed. The problem is to estimate (or predict) simultaneously $\theta_1,\ldots\theta_n$ or a subset $\theta_i,\theta_{i+1},\ldots,\theta_n$ or just θ_n of the current observation, given $y_1,\ldots,y_n$. There are various approaches to the problem depending on the purpose for which estimates are required and the information available on the joint distribution of (θ,y).

Submitted: *Advances in Biometry.*

95-25 Zhu-Yu Li, Wen-Yang Zhang, Gen-Xiang and Shailaja Suryawanshi. AN ESTIMATING OF ERROR DISTRIBUTION IN NONPARAMETRIC REGRESSION.

For a nonparametric regression model $y_i=g(x_i)+e_i$, $i=1,2,\ldots n$, $g(\circ)$ is assumed unknown and is to be estimated based on two situations: x_i are controlled and fixed without noise or (x_i,y_i) are i.i.d. observations of random variable (X,Y). Suppose e_i come from an unknown density function f(x). In this paper, we will establish the large sample properties and the asymptotic normality for the Rosenblatt estimator of f(x) based on N-W kernel estimator and a modified G-M border kernel estimator of g, by assuming some mild conditions on bandwidth parameters. Some simulation results will be given, and the smoothness among these estimators will be compared through graphs.

Submitted: *J. of Nonparametric Statistics.*

95-26 A.A.G. Alharbi, D.N. Shanbhag and L. Thabane. SOME STRUCTURAL PROPERTIES OF THE BHATTACHARYYA MATRICES.

In this paper, we study the structure of the 2x2 Bhattacharyya matrix relative to an exponential family and identify, amongst other things, the situations under which this matrix is diagonal or has polynomial nondiagonal elements (or is independent of the parameter). We also identify here the families for which the (1,1)th or (1,2)th element of the respective Bhattacharyya matrix is proportional to the corresponding element of a prescribed Bhattacharyya matrix.

Submitted: *J. of Applied Prob.*

95-27 C.R. Rao, V.K. Srivastava and H. Toutenburg. PITMAN NEARNESS COMPARISONS OF STEIN-TYPE ESTIMATORS FOR REGRESSION COEFFICIENTS IN REPLICATED EXPERIMENTS.

This paper presents a comparative study of the performance properties of one unbiased and two Syein-type estimators for combining the estimates of coefficients in a linear regression model when data sets are available from

replicated experiments conducted at possibly different stations.

Submitted: *Statistical Papers* (Germany).

95-28 H. Toutenburg and Shalabh. PREDICTIVE PERFORMANCE OF THE METHODS OF RESTRICTED AND MIXED REGRESSION ESTIMATORS.

This article considers the problem of simultaneous prediction of actual and average values of the study variable in a linear regression model when a set of linear restrictions binding the regression coefficients is available, and analyzes the performance properties of predictors arising from the methods of restricted regression and mixed regression besides least squares.

Submitted: *Biometrical Journal* (Germany), Accepted.

95-29 H. Toutenburg and V.K. Srivastava. EFFICIENCY PROPERTIES OF WEIGHTED MIXED REGRESSION ESTIMATION PROCEDURE.

This paper considers the estimation of coefficient vector in a linear regression model subject to a set of stochastic linear restrictions binding the regression coefficients and presents the method of weighted mixed regression estimation which permits to assign possibly unequal weighage to prior information in relation to sample information. Efficiency properties of this estimation procedure are analyzed when disturbances are not necessarily normally distributed.

Submitted: *Communications in Statistics - Theory and Methods.*

95-30 C.R. Rao and D.N. Shanbhag. RECENT APPROACHES TO CHARACTERIZATIONS BASED ON ORDER STATISTICS AND RECORD VALUES.

Recent advances on the integrated Cauchy functional equation have changed the status of the characterization theory of probability distributions significantly. The monographs of Ramachandran and Lau (1991) and Rao and Shanbhag (1994) provide the relevant details in this connection. In the present article, we restrict ourselves to discussing various characterization results based on order statistics and record values, and to unifying and improving these via techniques involving the integrated Cauchy functional equation or its variants. Our revelations show among other things that some of these results are linked with the characterization based on the strong memoryless property, of the exponential and geometric distributions. Also, we briefly touch upon here statistical applications of some of the results.
Submitted:

Appendix E

Participation in Conferences during the period 1993-95

C.R. Rao attended the following conferences and presented invited talks.

1993

1. Indo-US-Japan Conference on Quality through Engineering Design, January 11-14, Bangalore, India.
   Inaugural Address: Emerging technologies of the third millennium, January 12.
   Invited Talk: Some aspects of quality control methods, January 13.
   Panel Discussion on R & D Implications: Concepts and Methods, January 14.
2. Symposium on repeated measurements and cross-over designs, May 22, Temple University, Philadelphia.
   Invited Talk: Multivariate linear model with latent variables: Problems of estimation, May 22.
3. 49 Session of the International Statistical Institute, Firenze August 25-Sept.2
   President's Invited Talk: Statistics must have a purpose: The Mahalanobis dictum, August 27.
4. Statistics and Probability Symposium, September 20-October 2. Kent State University.
   Keynote Address: Measures of diversity and applications: A review of recent results, September 30.

1994

5. The Fifth Tartu Conference on Multivariate Statistics, May 23-28, Piihajarve, Estonia.
   Invited talk: The use of Hellinger distance in graphical displays of contingency table data, May 23.
6. Probastat'94: Conference on Regression Models, May 30-June 3, Smolenice, Slovakia.
   Invited opening address: Recent contributions to censored regression models, May 30.
7. 28-th Annual Asilomar Conference on Signals, Systems, and Computers, Oct.31-Nov.2.
   Invited talk: Some statistical problems associated with signal processing, Oct.3.
8. Conference on Environmetrics, Burlington, Canada.
   A special invited talk: Canonical coordinates: An alternative approach to correspondence analysis.
9. Seminar of Genetic Epidemiology and XX Annual Conference of Indian Society of Human Genetics, Dec. 11-13, 1994, Hyderabad, India.
   Inaugural address: Modern trends of research in human genetics, Dec. 11.
10. 3rd International Conference on DNA Fingerprinting, Dec. 13-16, 1994.
    Discussion on various topics.
11. Golden Jubilee Celebration of the Department of Statistics, Presidency College, Calcutta and National Seminar on Statistics - Its Social Relevance, Dec. 24-25, 1994.
    Inaugural address: The importance of statistics in national affairs, Dec. 24.

12. Second International Triennial Calcutta Symposium on Probability and Statistics, Dec. 30, 1994 - Jan. 2, 1995.
Invited talk in R.C. Bose Memorial Session: The use of Hellinger distance in graphical display of contingency table data, Dec. 30

1995

13. R.C. Bose Memorial Conference on Statistical Design and Related Combinatorics,, Colorado State University, 7-11 June 1995.
Special invited talk: Minimal row-column designs, June 9.
14. Fourth International Workshop on Matrix Methods in Statistics, Montreal July 15-16, 1995.
Invited talk: Some new ways of dealing with multicollinearity in a regression problems, July 16.
15. International Conference on Statistical Methods and Statistical Computing for Quality and Productivity Improvement, Seoul, Korea, Aug 17-19, 1995.
Keynote Address: Data Analysis and Statistical Thinking for Quality and Productivity Improvement (with T.S. Arthanari). Paper presented by Bovas Abraham, Waterloo University, Canada.